Mass and Interactions of the W Meson*

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Some possible sources of a large mass for the weak vector meson, or W meson, are studied. Several possibilities are found, including the following: (1) The self-mass of the W, which cannot be zero, is large. (2) The weak interactions generate a large self-mass. (3) The electromagnetic interactions generate a large self-mass. (4) The W undergoes a strong self-interaction which generates a large self-mass. Some experimentally observable consequences of these possibilities are discussed.

I. INTRODUCTION

SOME preliminary evidence in favor of a semiweakly coupled meson, the W meson, has recently been reported by the CERN neutrino experimenters.[1] This group reports that the mass of the W meson is at least 1.3 BeV.

If we provisionally accept this report, it is natural to enquire about the origin of the large mass of the W meson, a particle which, as we shall see, does not have any of the conventional strong interactions of the baryons and other mesons. When we say the W mass is large, we mean compared to the electromagnetic mass differences of 1 MeV, or the weak K_1-K_2 mass difference of 10^{-5} eV. In this paper, we shall consider several possible origins for the W mass, and their connection with yet unrevealed properties of the W meson. Some of the models we consider lead to new experimental effects, which should not be difficult to detect once the W meson can be produced in significant quantities. We do not, in this note, obtain numerical answers for the mass in any of the models considered.

II. FIELD EQUATIONS OF THE W MESON

Let us consider first the equations satisfied by the W field in the presence of weak interactions. We write the Lagrangian of the interacting fields as follows[2]:

$$\mathfrak{L}=-\tfrac{1}{2}(\partial_\mu W_\nu^*-\partial_\nu W_\mu^*)(\partial_\mu W_\nu-\partial_\nu W_\mu) -m_0^2W_\mu^*W_\mu+g(J_\mu W_\mu^*+J_\mu^*W_\mu)+\mathfrak{L}_0'. \quad (2.1)$$

Here $\mathfrak{L}_0'$ is the Lagrangian of the fields interacting with W_μ, m_0 is the *bare mass* of the W meson, and the vector J_μ is the current source of the vector meson field. For example, if we consider only the interaction of e and ν_e with W, then

$$J_\mu=i\bar\psi_e\gamma_\mu(1+\gamma_5)\psi_{\nu_e}. \quad (2.2)$$

The star notation is defined by

$$J_\mu^*\equiv(J_i^+,\,-J_4^+)=i\bar\psi_{\nu_e}\gamma_\mu(1+\gamma_5)\psi_e,$$
$$W_\mu^*\equiv(W_i^+,\,-W_4^+).$$

For the time being, we consider the interaction defined by Eqs. (2.1) and (2.2). The field equations obtained by varying (2.1) are

$$\partial_\mu(\partial_\mu W_\nu-\partial_\nu W_\mu)-m_0^2W_\nu+gJ_\nu=0. \quad (2.3)$$

From (2.3) it follows immediately that

$$m_0^2\partial_\nu W_\nu=g\partial_\nu J_\nu. \quad (2.4)$$

This is a familiar equation for vector meson fields. For free fields, or for fields interacting with a conserved current, the right-hand side of (2.4) vanishes, and this equation enables one to express the unphysical scalar field in terms of the spin-one fields.

In the present case, however, the weak interaction current J_μ is not conserved. This is easily seen for the current (2.2), even if we neglect the electron mass. To see it, we use the field equations of electron and neutrino following from (2.1) and (2.2):

$$\partial_\mu\bar\psi_e\gamma_\mu+ig\bar\psi_{\nu_e}\gamma_\mu(1+\gamma_5)W_\mu=0, \quad (2.5)$$

$$-\gamma_\mu\partial_\mu\psi_{\nu_e}+ig\gamma_\mu(1+\gamma_5)\psi_eW_\mu=0. \quad (2.6)$$

These give

$$\partial_\mu J_\mu=2ig[\bar\psi_e\gamma_\mu(1+\gamma_5)\psi_e-\bar\psi_{\nu_e}\gamma_\mu(1+\gamma_5)\psi_{\nu_e}]W_\mu. \quad (2.7)$$

We see that in the presence of weak interactions, the current (2.2) is not conserved. Generalization of J_μ to include muons, baryons, etc., will not change this. Of course, J_μ is conserved for massless electrons when the weak interactions are neglected. Hence in this approximation $\partial_\mu J_\mu$ is proportional to g.

One might suppose that by adding suitable extra terms to J_μ, W-W scattering terms for example, it would be possible to restore the conservation of J_μ. The author has been unable to find any way of doing this, so long as charged W mesons alone are present. Indeed, conserved currents are usually associated with invariances of the theory, and the theory of charged W mesons interacting with leptons has no invariance which would be generated by a charged current. If one wishes to extend the theory to obtain such an invariance, it appears necessary to introduce neutral W

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[1] Report on CERN neutrino experiment, presented by D. Perkins at the International Conference on Cosmic Ray Physics, 1963 (unpublished).
[2] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

mesons interacting with leptons. One could then envisage a theory of the type of Yang and Mills,[3] which *would* have the W's interacting with conserved currents. However, the absence in weak interactions of neutral lepton currents apparently precludes this.

Even if we neglect the weak interactions in the calculation of $\partial_\mu J_\mu$, this will not vanish when we include the finite fermion mass, as well as other terms in J_μ, such as the baryon axial vector current. In the discussion above, we have emphasized the weak contribution to $\partial_\mu J_\mu$, as that seems sure to be present in any theory resembling experiment. But it may well be that the other nonweak terms in $\partial_\mu J_\mu$ are more important for determining m_0, as they are independent of the small coupling constant g.

If we accept that $\partial_\mu J_\mu \neq 0$, we see that Eq. (2.4) tells us that m_0, the bare mass of the vector meson, cannot be equal to zero. This is a situation which to the author's knowledge has not occurred previously in physics, i.e., the field equations restrict the value of one of the parameters appearing in them. It is of course tempting to conclude that the large observed mass of the W arises from a large bare mass. We shall record this as the first possibility for the origin of the W mass.

Possibility 1. The bare mass m_0 of the W meson is quite large, and makes up almost all of the observed mass.

In order to investigate this possibility, we would like to use Eqs. (2.4) and (2.7) generalized to the case when J_μ is the total charged weak current, to calculate m_0^2. We do not know if this can be done. However, there are two simple features of Eqs. (2.4) and (2.7) worth noting.

The first point is that in the approximation considered above, m_0^2 appears to be proportional to the squared weak coupling constant. This might lead us to believe that m_0^2 is a small number. Such a conclusion may be unwarranted, as mass operators are usually divergent quantities in perturbation expansion, and for such objects, we know that the actual dependence on coupling strength may be quite different from the apparent one.[4]

Secondly, Eq. (2.4) seems to give an expression for m_0 in terms of quantities characterizing the other particles with which the W interacts. This may be of great importance in setting the scale for the W mass in terms of other physical masses. In particular, the baryon axial vector current may be of decisive importance in this connection, as its divergence is currently supposed to be proportional to the physical mass of the pion.

It should also be remarked that Eq. (2.4) should also contain terms on the right-hand side coming from electromagnetic interactions of W. These also generate a nonconserved current source for the W field, except perhaps for special values of the W magnetic moment. The author has no firm opinion as to the relative importance of electromagnetic terms and weak terms for the determination of m_0. It would obviously be of great interest to develop a calculation scheme for extracting the value of m_0 from Eq. (2.4) together with the expression for the current source J_μ.

III. WEAK AND ELECTROMAGNETIC SELF-MASSES OF W

If the charged W meson exists at all, it surely undergoes weak and electromagnetic interactions. It is natural to look to these interactions as possible sources of a large self-mass for the W. Of course, if one calculates the self-masses in perturbation theory, one obtains divergent expressions. The usual approach to such a calculation has been to cut off the divergent integrals at momenta corresponding either to the nucleon mass, or to the physical W mass. Since in lowest order one has expressions like

$$\delta m^2 \sim g^2 M^2, \quad \text{or} \quad \delta m^2 \sim e^2 M^2, \tag{3.1}$$

where M is the cutoff, it is clear that such a procedure necessarily leads to a self-mass which is only a small fraction of the "observed" mass of 1.3 BeV.

In spite of this, we wish to take seriously the possibility that the large W mass does come from its weak or electromagnetic couplings. In order to avoid the apparent divergences associated with the self-mass in perturbation theory, it is of interest to apply the technique for obtaining finite answers by summing up on infinite series of divergent graphs, which has been given the name peratization.[4] In this paper, we shall not actually carry out a summation, but instead use the so-called power-counting method[4] to indicate how a large self-mass could be obtained from weak or electromagnetic interactions. We note that if it is possible to peratize the W self-mass, we have gone beyond the standard renormalization theory. Even in the ξ-limiting formalism of Lee and Yang,[5] the W self-mass is infinite, and must be renormalized. But if we are correct in believing that δm^2 is finite when an infinite number of graphs are summed, it would indicate

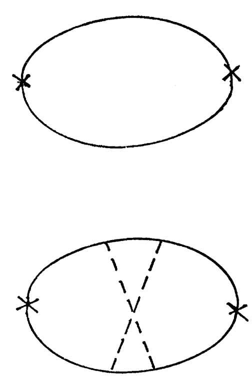

FIG. 1. Some of the graphs contributing to $\pi_{\mu\nu}$, coming from weak interactions. The crosses indicate external W's. Solid lines are leptons, dashed lines are W's.

[3] C. N. Yang and R. Mills, Phys. Rev. **96**, 191 (1954).
[4] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963).
[5] Reference 2 and T. D. Lee, Phys. Rev. **128**, 899 (1962).

that even in renormalizable theories such as quantum electrodynamics, it may be possible to compute quantities such as the electron self-mass by an extension of peratization.

Let us review some formal properties of the self-energy part and propagator for vector mesons. We define the self-energy part $\Pi_{\mu\nu}$ as the sum of all proper two-point W graphs with legs removed (Fig. 1).

The relation between $\Pi_{\mu\nu}$, and the unrenormalized propagator $\Delta_{\mu\nu}'$, is given by

$$\Delta_{\mu\nu}'=\Delta_{\mu\nu}+\Delta_{\mu\tau}\Pi_{\tau\rho}\Delta_{\rho\nu}'. \quad (3.2)$$

Here $\Delta_{\mu\nu}$ is the bare propagator, which in momentum space has the form

$$\Delta_{\mu\nu}(p)=-i\frac{\delta_{\mu\nu}+(p_\mu p_\nu/m_0^2)}{p^2+m_0^2}. \quad (3.3)$$

We can use Lorentz invariance to write

$$\Pi_{\tau\nu}=\alpha(p^2)\delta_{\tau\nu}+\beta(p^2)p_\nu p_\tau. \quad (3.4)$$

In terms of this α, β, we obtain for Δ'

$$\Delta_{\mu\nu}'=\frac{-i}{p^2+m_0^2+i\alpha(p^2)}\left[\delta_{\mu\nu}+\frac{p_\mu p_\nu(1-i\beta)}{i\beta p^2+m_0^2+i\alpha(p^2)}\right]. \quad (3.5)$$

It follows from (3.5) that the self-mass depends only on $\alpha(p^2)$, and is given by the implicit equation

$$\delta m^2=m^2-m_0^2=i\alpha(-m^2), \quad (3.6)$$

where m^2 is the physical mass, defined by the pole in $\Delta_{\mu\nu}'$. We can use this to define a new function $\alpha_1(p^2)$, by

$$\alpha(p^2)\equiv\alpha(-m^2)+(p^2+m^2)\alpha_1(p^2) \quad (3.7)$$

and then obtain

$$\Delta_{\mu\nu}'=\frac{-i}{(p^2+m^2)(1+i\alpha_1)}\times\left[\delta_{\mu\nu}+\frac{p_\mu p_\nu(1-i\beta)}{ip^2(\alpha_1+\beta)+m^2(1+i\alpha_1)}\right]. \quad (3.8)$$

We note that $1+i\alpha_1(-m^2)$ is to be identified with the wave function renormalization Z_3^{-1}.

In Eq. (3.6) we express the self-mass δm^2 in terms of the self-energy part evaluated for particles propagating with the physical mass m. This is a part of the well-known procedure of mass renormalization. It is important to realize that this particular aspect of mass renormalization has nothing to do with the fact that self-energy integrals are divergent, but rather must be done to apply the standard perturbation expansion. It is also possible to obtain a series for δm^2 in terms of m_0^2, but that is less useful for our purposes.

Let us then estimate the contribution to $\alpha(-m^2)$ coming from weak and electromagnetic interactions.

A. Weak Interaction Contribution to the Self-Mass

The dependence of a weak interaction graph in $\Pi_{\mu\nu}$ on g and the cutoff is easily estimated, at least for intermediate states containing leptons and W mesons alone. In doing this, we introduce a cutoff on the lepton propagators, as described in FPII.[6] Furthermore, we take the W to propagate with the physical mass, and do not explicitly consider the fact that the W is unstable, i.e., that $\alpha(-m^2)$ is complex. In the opinion of the author, this is unlikely to cause serious complications in a field theory with a primitive W field.

We obtain as the contribution of a graph containing n vertices

$$\alpha_0^{(n)}\equiv\alpha^{(n)}(-m^2)=m^2g^n(M/m)^na_n, \quad (3.9)$$

where, as usual, M is the cutoff and a_n is a numerical coefficient, independent of g and M. This result is a simple extension of the power-counting method given previously. It is also worth noting that the momentum-dependent terms involving α_1, β are less divergent, and are given by expressions of the following type (see FPII, Appendix 2)

$$\begin{aligned}\alpha_1=\alpha_1^{(n)}(p^2=0)&=b_ng^n(M/m)^{n-2} && \text{for} \quad n>2\\ &=b_2g^2\ln(M/m) && \text{for} \quad n=2,\\ \beta_0=\beta^{(n)}(p^2=0)&=c_ng^n(M/m)^{n-2} && \text{for} \quad n>2\\ &=c_2g^2\ln(M/m) && \text{for} \quad n=2.\end{aligned} \quad (3.10)$$

In all of the above, we have written only the "most divergent terms." The remaining terms will hopefully add up to something containing more powers of g^2.

We can now carry out the sum of α_0, α_1, β_0 over all graphs. We obtain

$$\begin{aligned}\alpha_0&=m^2F(x),\\ \alpha_1&=b_2g^2\ln(1/g)+g^2G(x),\\ \beta_0&=c_2g^2\ln(1/g)+g^2H(x),\end{aligned} \quad (3.11)$$

where

$$\begin{aligned}F(x)&=\sum a_nx^n,\\ G(x)&=b_2g^2\ln x+g^2\sum b_nx^{n-2},\\ H(x)&=c_2g^2\ln x+g^2\sum c_nx^{n-2},\end{aligned} \quad (3.12)$$

and $x=gM/m$.

If we now make the usual assumption of power counting, that the limit functions $F(\infty)$, $G(\infty)$, $H(\infty)$ exist and are finite, we may conclude the following.

The weak interaction contribution to the W self-mass is in leading order independent of the weak coupling g. It is given by the relation

$$\delta m^2=im^2F(\infty)+O(g^2). \quad (3.13)$$

Since $F(\infty)$ is a number which is imaginary and presumably of a magnitude of order unity, this leads to a second possibility for the origin of the large W mass.

[6] G. Feinberg and A. Pais, Phys. Rev. **133**, B477 (1964). We refer to this paper as FPII.

Possibility 2. The W mass comes mostly from weak interactions, the divergent perturbation graphs summing up to a finite result.

In order to make this possibility any more than a speculation, we must be able to calculate $F(\infty)$. This will be taken up on another occasion. Suppose, however, that the self-mass *is* a mostly weak effect. We can then ask about other terms in the propagator. We see from (3.11), that the terms α_1, β_0 are small numbers, of order $g^2 \ln g$. Hence, so long as we consider momenta p for which $gp/m \ll 1$, it follows from (3.8) and (3.11) that the W propagator has approximately the momentum dependence of the bare propagator, except that it propagates with the physical mass. It may be feasible to test this experimentally, by measurement of weak interactions at large momentum transfer, say in neutrino-absorption experiments. The problem of distinguishing propagator corrections from other higher order corrections will be discussed further below.

B. Electromagnetic Contribution to the Self-Mass

The interaction of W with photons is also an unrenormalizable theory. A prescription for dealing with the divergences of this theory has been given by Lee.[5] In the procedure of Lee, the electromagnetic self-mass of the W is still infinite, in the sense that individual graphs are still divergent. However, in line with our previous analysis, we may conjecture that these infinities also sum to a finite answer, so that the self-mass may be calculable. It is therefore of interest to estimate the magnitude of the electromagnetic contribution to the self-mass. To do this, we first introduce a cutoff strong enough to make all graphs finite, which we again call M.

It is known from the work of Lee,[5] and of Bernstein and Lee[7] that vector meson electrodynamics is a rather different theory when the anomalous magnetic moment K of the vector meson is zero, than when it is nonzero.

Although both theories can be made to give finite answers by the method of Lee, the answer depends critically on K. In Ref. 7, an argument has been given in favor of the value $K=0$. Let us then consider this theory first.

The striking feature of vector meson electrodynamics with $K=0$ is that most Feynman graphs are much less divergent than a simple power count would indicate.[5,7] Furthermore, we find that for a graph in $\Pi_{\mu\nu}$ containing n powers of e, the most divergent terms are not independent of momentum, as in Sec. IIIA, but instead take the form

$$\Pi_{\mu\nu}{}^{(n)}(p)=e^n[A_n(p^2+m^2)(M/m)^n\delta_{\mu\nu} +m^2C_n(M/m)^n\delta_{\mu\nu}+B_n(M/m)^n p_\mu p_\nu] +\text{less divergent terms}. \quad (3.14)$$

Here A_n, B_n, C_n are numerical coefficients independent of p, M, or e. We see that for this case, the contributions to α_0, α_1 and β_0 are equally divergent.[8]

If we sum $\Pi_{\mu\nu}{}^{(n)}$ over all graphs, and pass to the limit $M\to\infty$, we get

$$\Pi_{\mu\nu}=(p^2+m^2)\delta_{\mu\nu}S(\infty)+m^2\delta_{\mu\nu}R(\infty) +p_\mu p_\nu T(\infty)+\cdots, \quad (3.15)$$

where

$$S(\infty)=\lim_{x\to\infty}\sum A_n x^n,\quad R(\infty)=\lim_{x\to\infty}\sum C_n x^n,\quad T(\infty)=\lim_{x\to\infty}\sum B_n x^n, \quad (3.16)$$

and

$$x=eM/m.$$

Hence, the leading electromagnetic contributions to $\Pi_{\mu\nu}$ at low momenta take the form:

$$\alpha_0=m^2R(\infty)\equiv m^2R,\quad \alpha_1=S(\infty)\equiv S,\quad \beta_0=T(\infty)=T, \quad (3.17)$$

all of which are independent of e.

Upon substitution into (3.6) and (3.8) we find

$$\delta m^2=im^2R, \quad (3.18)$$

$$\Delta_{\mu\nu}'=\frac{-i}{(p^2+m^2)(1+iS)} \times\left[\delta_{\mu\nu}+\frac{p_\mu p_\nu(1-iT)}{ip^2(S+T)+m^2(1+iS)}\right]. \quad (3.19)$$

From Eq. (3.18), we are led to a new possibility for the origin of the W mass.

Possibility 3. The W mass comes mostly from electromagnetic interactions, the divergent perturbation graphs summing up to a finite result.

One may be inclined to consider possibility 3 more likely than possibility 2, since the momentum at which electromagnetic interactions "become strong" is given by $p\sim m/e\sim 10$ BeV, whereas the weak interactions do not become strong until momenta $p\sim m/g\sim 300$ BeV. This argument may or may not appear compelling. The results presented here indicate that the weak interactions, and the electromagnetic interactions for $K=0$, are equally divergent, in the sense that adding two additional weak vertices to a graph produces the same change in the power of the cutoff as does the addition of two electromagnetic vertices. It is therefore necessary to treat the higher order weak interactions and electromagnetic interactions simultaneously here, as well as for other processes. Since this involves a

[7] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963).

[8] For this case, we have not looked into the question of possible powers of $\ln M$ in (3.14). In the corresponding expressions (3.10), there are no powers of $\ln M$ in leading order.

power series in the cutoff and two parameters g, e, it will require other techniques than the power count used previously. This subject will be taken up again elsewhere.

If possibility 3 should prove correct, we may expect some other effects to show up. In particular, if the quantities S, T defined above are nonzero, it follows from (3.19) that there are momentum-dependent corrections to the W propagator which should be observable at low momenta. These corrections occur in particular in the longitudinal term $p_\mu p_\nu$. It is important to recognize that the effect of such corrections cannot arise from higher order weak interactions, at least insofar as we are interested in terms observable at low momenta. This is because the leading term in the sum of higher order weak corrections is generally independent of momentum.[4] It is also necessary to examine electromagnetic corrections to the W-lepton vertex, etc., since the quantity $\Pi_{\mu\nu}$ is gauge-dependent, and only the sum of all electromagnetic corrections of a given order is invariant. While this has not been done in detail, there do not appear to be any large terms coming from other electromagnetic corrections which would compensate the indicated terms in $\Pi_{\mu\nu}$. One may therefore hope that by examining μ decay, W production by ν, or some other process unaffected by strong interactions, the existence of large corrections to the propagator could be observed.

Finally, a word concerning the theory with $K\neq0$. In this case, the reduction of divergence alluded to earlier does not occur, and the leading divergences take the form:

$$\Pi_{\mu\nu}^{(n)}(p)=f^n[m^2D_n(M/m)^{2n}\delta_{\mu\nu}+p^2E_n(M/m)^{2n-2}\delta_{\mu\nu}+J_n(M/m)^{2n-2}p_\mu p_\nu]+\text{less divergent terms} \tag{3.20}$$

with $f=eK$.

We see from this that the self-mass terms are now more divergent by two powers than the momentum-dependent terms. The expansion parameter is now

$$y=f^{1/2}M/m. \tag{3.21}$$

Upon going through the sum, we find

$$\alpha_0=m^2D(\infty),\quad \alpha_1=fE(\infty),\quad \beta_0=fJ(\infty). \tag{3.22}$$

Hence, in this theory, we again find that the self-mass is independent of the coupling constant, and may be the origin of the large observed mass. However, in the present case, the other propagator corrections have additional factor $f=eK$. Since e is not really small, it may nevertheless be possible to measure such corrections.

It is worth remarking that if the approach outlined in this section should prove correct, it would be sensible to reconsider the self-mass problem for other particles, such as the leptons. While the weak and electromagnetic contributions to the lepton self-mass are also divergent term by term, it could well be that the series sums to a finite answer. In that case one could envisage that also for these particles, the field theories are convergent, and only the previous methods of calculation are unreliable.

IV. OTHER POSSIBLE INTERACTIONS OF THE W MESON

If the W exists, then in addition to the weak and electromagnetic interactions considered in Sec. III, it could have other interactions which might be relevant to the mass. It is not difficult to show that if the W mass is indeed on the order of 1.3 BeV, then it does not have strong interactions with baryons or other mesons. To see this, we consider several interactions in turn.

A. Strong Interactions of W with Baryons or Pions, Etc.

1. Interactions Linear in W and Bilinear in Other Fields

Such interactions would be similar in structure to the weak interactions of W. They would therefore contribute to such reactions as β decay. But the β decay coupling constant is comparable to the μ decay constant, to which the interactions 1 do not contribute. We conclude that any interactions of type 1 cannot be much stronger than the interaction of the W with the weak baryon current. This interaction has a strength given approximately by

$$g\sim(G_Fm_w^2)^{1/2}\sim3\times10^{-3}(m_w/m_p). \tag{4.1}$$

Hence if $m_w\sim m_p$, we may conclude that any interactions of type 1 must have a rather small coupling constant.

2. Interactions Quadratic in W and Bilinear in Other Fields

Interactions of this type were recently proposed by Ericson and Glashow.[9] Suppose that in the CERN experiment, 1.3-BeV mass W's are really being produced, with the indicated cross section of $\sim10^{-39}$ cm^2. We can then conclude that any interaction of type 2 is restricted in size by the approximate inequality

$$c\lesssim e^2/m_n, \tag{4.2}$$

where c is the coupling constant.

To see this, we note that in the process considered at CERN

$$\nu+p\rightarrow\mu+W+p, \tag{4.3}$$

the W is produced by a weak interaction, and then must scatter from the nucleon to conserve momentum. It is usually assumed that this scattering occurs by the

[9] T. Ericson and S. L. Glashow, Phys. Rev. **133**, B130 (1964).

Coulomb field, leading to an over-all matrix element

$$M \propto ge^2/q^2, \tag{4.4}$$

where q is the momentum transferred to the proton.

If an interaction of the form 2 exists, with a strength c, the W could instead scatter from the proton through it, leading to a matrix element

$$M' \propto gc/m_w. \tag{4.5}$$

Since the observed cross section is surely no larger than that given by (4.4), we are safe in concluding that

$$c < e^2 m_n/q^2,$$

which for the conditions of the experiment effectively gives (4.2).

An interaction of that size can hardly be considered strong, and in any case could be generated through the electromagnetic interaction of the W. It therefore seems justified to conclude that for a W of the assumed mass, no anomalously strong interactions of type 2 exist.

Similar arguments may be carried through for W-baryon or W-pion interactions in which more fields interact at a point, all leading to the conclusion that the W has only weak and electromagnetic interactions with the strongly interacting particles.

B. Possible Strong Self-Interaction of W

There is another type of interaction which the W might have, which apparently would not have shown up in experiments done until now, even if it were strong. We refer to a strong self-interaction of the W, involving four, or conceivably more, mesons at a point. Since most of the effects we discuss will not depend qualitatively on the number of mesons interacting at a point, or upon the momentum dependence of the interaction, we shall for simplicity deal with a four-meson interaction of the form

$$aW_\mu{}^*W_\mu W_\nu{}^*W_\nu + bW_\mu{}^*W_\nu W_\mu{}^*W_\nu, \tag{4.6}$$

with a, b constant.

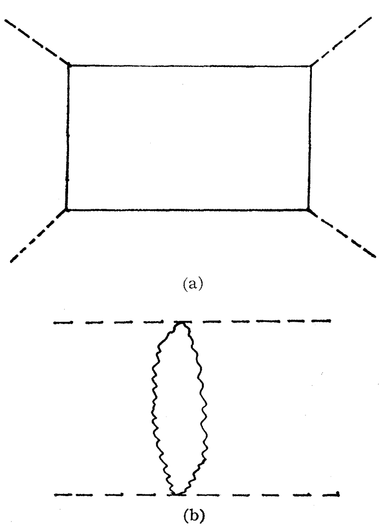

FIG. 2. (a) A meson-meson scattering via weak interactions. Solid lines are leptons, dashed lines are W's. (b) A meson-meson scattering via photons. Wavy lines are photons, dashed lines are W's.

There is at least one possible origin for such a term. This is the application of the standard renormalization theory to the ξ limiting formalism of Lee and Yang.[5] In that formalism, the W propagator is regularized in such a way that for finite values of the regulator mass ($M=\xi^{-1/2}m$), the theory is as divergent as that of a spin-zero meson. This still leaves some divergences, and in particular even for finite M the meson-meson scattering graphs such as Fig. 2 are divergent.

Lee[5] has suggested that these graphs should be renormalized, by the introduction of a direct meson-meson scattering counter term

$$(\lambda_1+\delta\lambda_1)W_\mu{}^*W_\mu W_\nu{}^*W_\nu + (\lambda_2+\delta\lambda_2)W_\mu{}^*W_\nu W_\mu{}^*W_\nu. \tag{4.7}$$

Here $\delta\lambda_1$, $\delta\lambda_2$ are chosen to cancel the divergent parts of Fig. 2, which are easily seen to be momentum-independent. The terms with λ_1, λ_2 are the residual meson-meson interactions, of the form (4.6).

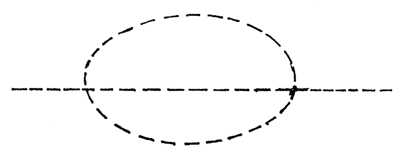

FIG. 3. A graph contributing to the W propagator, coming from the interaction (4.6). Dashed lines are W's.

The point we wish to make is that the magnitudes of λ_1, λ_2 are completely undetermined by the renormalization prescription, and instead are new physical constants which must be obtained from experiment. In particular, there does not appear to be any reason why λ_1, λ_2 should be small, even though $\delta\lambda_1$, $\delta\lambda_2$ are proportional to four powers of e or g. Let us therefore analyze the possibility that λ_1, λ_2 and hence a, b are numbers of order 1, and see where it leads us.

A similar situation of course exists for the pion-nucleon interaction, for which case the meson-meson scattering is also divergent, and must be renormalized. It is recognized that the pion-pion scattering constant is independent of the pion-nucleon coupling constant, at least in the framework of renormalization theory. But for that case, since all couplings are large, no new qualitative feature emerges from this. In the present case, since the W has no other strong couplings, a large value for λ_1 or λ_2 would have striking consequences.

We therefore suppose that either from this source or some other, there is a meson-meson interaction like (4.6) with either a or b or both of order 1. Such a coupling would have a number of effects that should be experimentally accessible.

1. *Modification of the W Propagator*

Evidently, a strong interaction (4.6) would contribute large terms to $\Pi_{\mu\nu}$ from graphs like Fig. 3, and hence to the W propagator. In particular, a large self-mass for W would not be at all surprising, since strong interactions are commonly supposed to produce large self-mass effects.

We are therefore led to another possible origin of the W mass.

Possibility 4. The W undergoes a strong self-interaction, which generates a large self-mass, forming the major part of the physical mass.

It will be difficult to obtain any quantitative results from possibility 4, as strong interactions are notoriously hard to deal with. However, if some of the other effects of this interaction are detected, the W mass problem would not be any more vexing than that of the baryons.

Graphs like Fig. 3 will give not only a self-mass, but also alter the momentum dependence of the propagator. In the present case, since there is no small parameter (except p/m), there is no reason to expect the functions α,β of (3.4) to be constant. Hence we may expect the

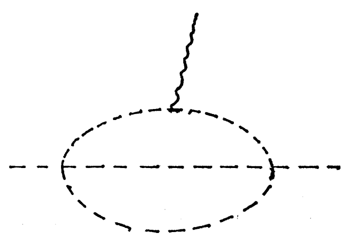

FIG. 4. A graph contributing to the W electromagnetic interaction, coming from the interaction (4.6). The wavy line is an external photon, dashed lines are W's.

ratio of the physical propagator $\Delta_{\mu\nu}'$, to the bare propagator $\Delta_{\mu\nu}$ to vary appreciably as the momentum varies between $p^2=0$ and $p^2=-m^2$, say. This would have several experimental consequences.

(a). The coupling constant g', measured in β decay, μ decay, and other low-energy processes would differ from the true renormalized coupling constant g, as measured in the W decay by a factor

$$\frac{g'}{g}=\frac{1+i\alpha_1(-m^2)}{1+i\alpha_1(0)}. \tag{4.8}$$

If α is not a slowly varying function, this could be appreciable. However, it should be cautioned that the corresponding factor for pions appears to be close to 1.

(b). The form factors in neutrino-absorption experiments will be modified because of the momentum dependence of $\Delta_{\mu\nu}'$. If we write

$$\Delta_{\mu\nu}'=A(p^2)\delta_{\mu\nu}+B(p^2)p_\mu p_\nu \tag{4.9}$$

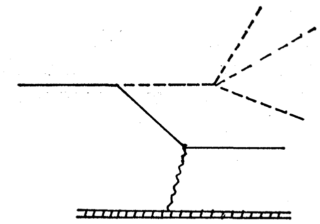

FIG. 5. A graph leading to multiple production of W's, via interaction (4.6). Solid lines represent leptons, dashed lines represent W's, the wavy line represents a Coulomb photon, and the double line a proton.

then we find,[10]

$$\begin{aligned} g_V'&=g_VA(p^2);\\ f_V'&=f_VA(p^2),\\ g_A'&=g_AA(p^2);\\ h_A'&=h_A[A(p^2)+p^2B(p^2)]-2mg_AB(p^2), \end{aligned} \tag{4.10}$$

where the primed functions are the observed form factors, and the unprimed ones are given by matrix elements of the baryon current in the one baryon state.

2. *Electromagnetic Interactions of the W*

In the presence of the interaction (4.6), the electromagnetic interactions of the W, such as magnetic moment, quadrupole moment, charge distribution, etc., will be modified, just as for the proton, pion, and other strongly interacting particles. In particular, we note that if (4.6) is a strong interaction, we would expect a quadrupole moment for the W, to be generated by graphs like Fig. 4, of order

$$Q\cong e/m_w^2.$$

On the other hand, in the absence of such a strong interaction, the quadrupole moment has been estimated[5] to be of order

$$Q\sim e/m_w^2\alpha\ln\alpha.$$

It will perhaps eventually be possible to distinguish between these cases. Similar results hold for the anomalous magnetic moment, which in the presence of (4.6), we would expect to be of order $eK\sim e/m_w$. In this case it is not clear what to expect for K in the absence of a strong interaction.

3. *Multiple Production of W's*

In any process, such as (4.3), where a real W can be produced, if the energy is sufficiently high there will be a finite probability that the W will convert into three W's via the interaction (4.6). One diagram through which this can occur is given in Fig. 5.

If (4.6) is a strong interaction, the rate for

$$\nu+p\rightarrow\mu+3W+p, \tag{4.11}$$

should be comparable to that for (4.3), once the energy

[10] Apart from the normalization of the form factors, we use the notation of T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).

is well above threshold. A signature of (4.11) would be an event in which more than two charged leptons emerge, which would occur if each W in (4.11) decays via one of its leptonic modes. For a W of mass 1.3 BeV, the threshold for (4.11) when the target is a heavy nucleus is about 4 BeV, so that it would be of interest to look for events with 3 or 4 charged leptons emerging in the present series of neutrino experiments.

4. *Final-State Interaction of W Pairs*

In a reaction producing a pair of W's, such as photoproduction, or

$$p+\bar{p}\rightarrow W^{+}+W^{-}+\pi^{0},$$

the outgoing pair could undergo a strong final-state interaction through (4.6). This would have the effect of changing the energy and angular distributions of the pair from those predicted by electromagnetic theory. Presumably, such an effect would be measurable in the proposed photoproduction or annihilation experiments.

5. *Possible W-W Resonances*

If W's indeed undergo strong self-interactions, the possibility exists of forming resonant or bound states of these particles. Such states might in principle compose as complicated a system of particles as do the pion resonances, but they would be coupled to the usual particles only by weak or electromagnetic interactions. It is interesting to imagine that a whole world of particles could exist, connected with the world of baryons, etc., only by the exchange of intermediate vector mesons. It would also be of interest to test this possibility experimentally.

In concluding this discussion of W-W interactions, we should note the following. While the graphs of Fig. 2 are divergent, it is possible that by summing up a series of such graphs, one obtains a finite result for W-W scattering. If this should happen there is no reason to renormalize the W-W scattering, and the strength of the interaction could be calculated, unless there is an additional "primitive" W^4 coupling. We can apply the previous power counting techniques to the W-W scattering via leptons. We find that for the amplitude (4.6)

$$a\sim O(g^4 \ln g),$$
$$b\sim O(g^4 \ln g), \qquad (4.12)$$

which are still small.

Similarly, for the amplitude via photon exchange, in the case $K=0$, we find

$$a\sim O(e^4 \ln e),$$
$$b\sim O(e^4 \ln e). \qquad (4.13)$$

Hence if it is possible to "peratize" the W-W scattering, we would not expect the effects described under 1–5 to occur. It may therefore be possible to distinguish between the possibilities of peratization or a strong renormalization constant soon experimentally.

V. SUMMARY AND CONCLUSIONS

We have found four possible explanations of the large physical mass of the W. Let us summarize what they are.

Possibility 1. The bare mass of the W, which cannot be zero, is quite large, and makes up almost all of the observed mass.

The main indication in favor of this possibility is that the bare mass of the W cannot vanish because of the nonconservation of the weak current. This possibility also seems to offer some hope of relating the W mass to the masses of the strongly interacting particles, since the divergence of the axial vector current depends on these masses critically.

Possibility 2. The W mass comes mostly from weak interactions, the divergent graphs summing up to a finite result.

If this possibility is the correct one, it should be possible to actually compute δm^2 at least for models. One would also then want to re-examine the question of self-masses in other, renormalizable, theories to see whether there too one can obtain finite answers by summation of divergences.

Possibility 3. The W mass comes mostly from electromagnetic interactions, the divergent perturbation graphs summing up to a finite result.

Much the same remarks hold here as for possibility 2, with the addition that in this case there may be other observable propagator effects.

In these first three cases, it is of course necessary to carry out explicit calculations to determine whether the behavior we have considered is really present. Hopefully, such behavior would show up in calculations with a restricted set of graphs, of the type which can now be done.

Possibility 4. The W undergoes a strong self-interaction, which generates a large self-mass, forming the major part of the physical mass.

In this case, it will not be easy to compute the W mass, but there should be many observable effects of the strong self-interaction.

It is to be hoped that the experimental situation regarding the existence of the W will soon be clarified, so that theorists can enter into such calculations with somewhat more justification.

ACKNOWLEDGMENTS

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